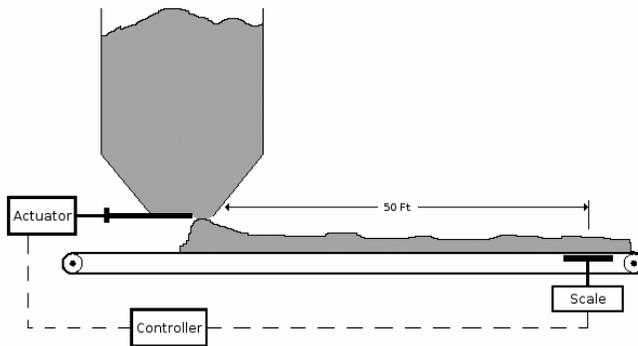


## Process Dynamics

Process dynamics refer to any number of time elements inherent in each device within the control loop and the process. These dynamics are expressed as a dead time and a time lag. Although each individual element within a system does have its own dead time and time lag, these elements combine mathematically to provide a system time lag and a system dead time. It is these two pieces of data in which we are interested.

### Dead Time

Process dead time is easiest of the two time elements to define, but the most difficult to control. Dead time is also referred to a transportation delay. When a control loop makes a corrective action to a process, the effects of this action are not realized within the feedback loop until sensed by the sensor element. The time it takes for this to happen is called dead time. Consider the conveyor belt system illustrated below. The belt is moving at a velocity of 15 fps. At the end of the conveyor belt, 50 ft away, is a scale measuring the weight of product released from the hopper. The controller is calibrated to change the position of the gate on the hopper, based on input from the scale, allowing the product to drop. However, at any time  $T_0$ , the amount of material actually released is not sensed by the scale until the conveyor belt moves the product to a point above the scale. Since the conveyor is moving at a finite velocity, the amount of time it takes for the material dropped at  $T_0$  is easily calculated as:



$$T_d = \frac{50 \text{ ft}}{15 \text{ fps}} = 3.33 \text{ sec}$$

In many cases, one can minimize the affects of dead time by placing the sensor properly. In the case of the conveyor belt, placing the scale closer to the hopper would greatly reduce dead time making the system easier to control. In theory, we could place the scale directly under the hopper and reduce dead time to

Figure 1: An illustration of dead time

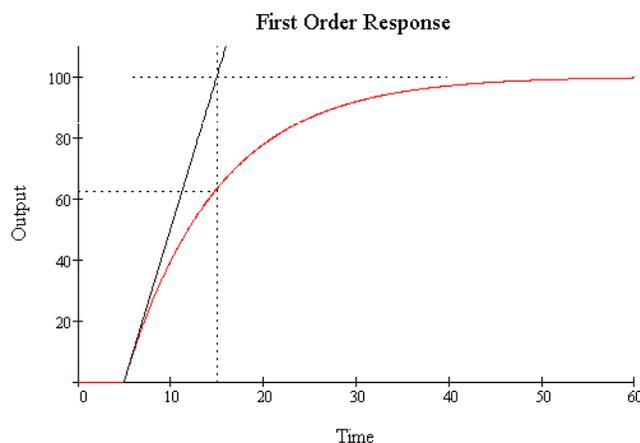
nearly zero. However, this would not be prudent. Doing so would cause the impact of the product to produce erroneous measurements. In other words, proper sensor placement is much more than a function of dead time. Based on this analogy, it becomes evident that there will always exist some amount of dead time.

Multiple dead times in a system are additive. If two components of a system are in series, one with a dead time of one second, the other with a dead time of two seconds, then the total dead time would be three seconds. Using the conveyor belt example, consider the fact the hopper is located some distance above the conveyor belt. As such, it takes time for the product to drop from the hopper to the belt. If this time is one-half second, then the dead time in the above process would be 3.83 seconds.

### ***First Order Time Lags***

Ideally, if the engineering value of a process variable changes, we would like the sensing element to react to this change immediately and synchronously. However, this is never the case. There will always be a lag in the sensing device. Similarly, we would like to see the movement of a valve actuator to exactly and immediately follow the output of the controller. Although there are devices that respond quickly to an external influence, no device responds instantaneously. There is always some form of lag. This lag is referred to as a time lag.

Consider the following scenario. An RTD type of temperature sensor is stabilized at room temperature, then suddenly immersed in a beaker of water at a temperature of 20 °F above the ambient air temperature. This would constitute a step change in input signal (temperature) to the RTD. Ideally, the output of the RTD would follow the input signal. However, the RTD has mass. As such, it takes time for the temperature of this mass to rise such that the resistance of the device changes. As shown in the figure below, the output signal will not rise immediately. First, a certain amount of time will pass before anything happens. This would be a dead time. Once the dead time has passed, the output of the RTD will increase slowly and at a variable rate. This results in response referred to as a first order lag plus dead time (FOLPDT). The terminology refers to the type of mathematical equation, a first order differential equation, used to describe this response.



The shape of this curve is described by a time constant. The time constant of a first order response is equal to the amount of time it takes to achieve 63.2% of the total output. The time constant can be determined graphically as indicated in the Figure 2. Assume the step function that generated this response occurred at time  $t=0$ . The procedure for finding the time constant is as follows.

*Figure 2: Graphic determination of time constant for first order lag*

- Draw a horizontal baseline tangent to the final value of the response curve (100%).
- Dead time occurs from time  $t=0$  to the point of initial response (5s).
- Draw a tangent line to the steepest part of the curve until it intersects the upper baseline. The steepest part of a true first order response always occurs at the end of the dead time.
- From the intersection of the tangent line to the upper baseline, draw a vertical line to the time axis. The difference between this time and the end of the dead time represents the time constant of this first order response. In Figure 2, the time constant is 10 seconds. Also note the vertical crosses the response curve at a point equal to 62.3% of the total change in output.

### **High Order Time Lags**

When multiple first order lags are placed in series, the response can be rather complex. The multiple time lags may be either interacting or non-interacting. This can be explained through a hydraulic analogy. Consider a group of series cascading tanks as arranged in Figure 3(a). This represents non-interacting lags. Assume the valves are partially open and the system is at steady state. The flow rate from each tank is dependent only on the fluid head in each tank and how much the valve is opened. If we open valve one further, the flow from tanks two and three are eventually affected. However, if we open valve two instead, this will impact tank 3 but not tank 1.

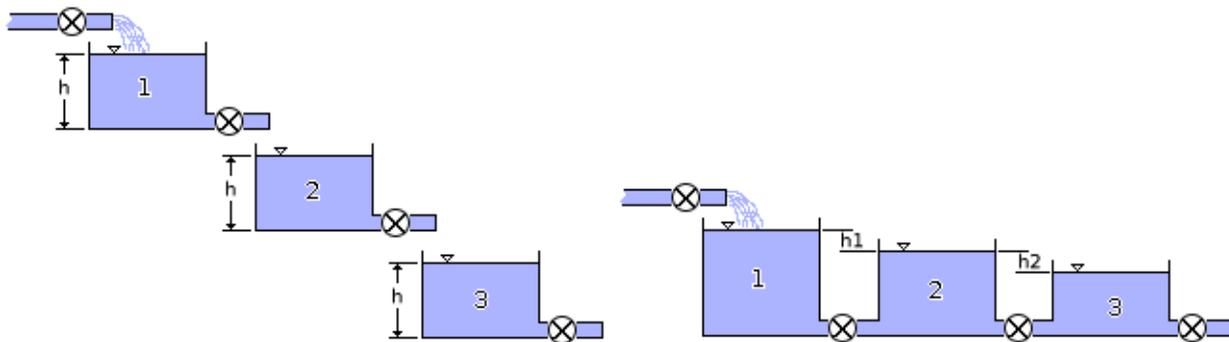


Figure 3: a) Noninteracting Lags

b) Interacting Lags

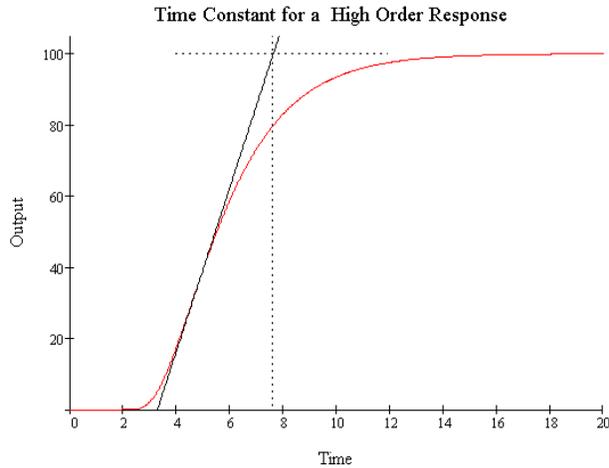
Figure 3(b) considers a similar arrangement. However, the flow rate from each tank is a function of the differential head between tanks. Any change in flow rate from one tank to the downstream tank not only impacts the downstream tank, but also the upstream tanks. Such an arrangement represents interacting lags.

Whereas a first order response is typical of a single component of the control system or process, a high order response is the result of several components of the system or

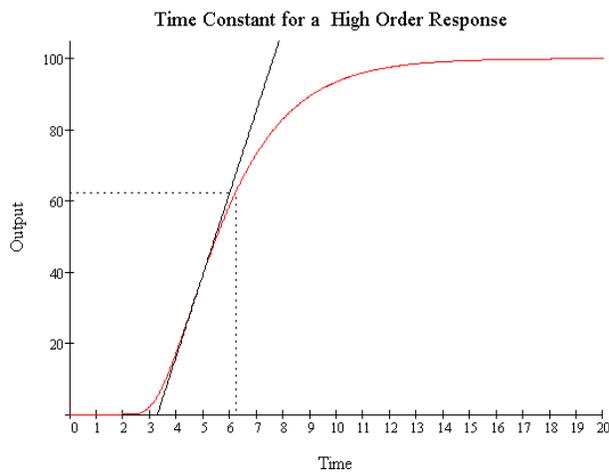
process in series. The traces in Figure 4 are representative of such a system. These traces were generated with six non-interacting time lags and a dead time. The step function producing this response occurs at time  $t=0$ . For our purposes, we need to analyze the response for an overall time lag and an overall dead time. We are not usually concerned with the number of time elements involved in the response or where they occur within the system. However, as we'll discuss later, it is often wise to understand where the dominant time lags and the dominant dead times occur within the system. The analysis of a high order response may be handled in one of the three ways described in Figure 4. Although all three methods described are accepted methods of analysis, they provide very different results.

Note that the first method tends to overestimate the time constant. In most cases, this is not desirable. However, this method has the advantage of being a very simple method of graphical analysis.

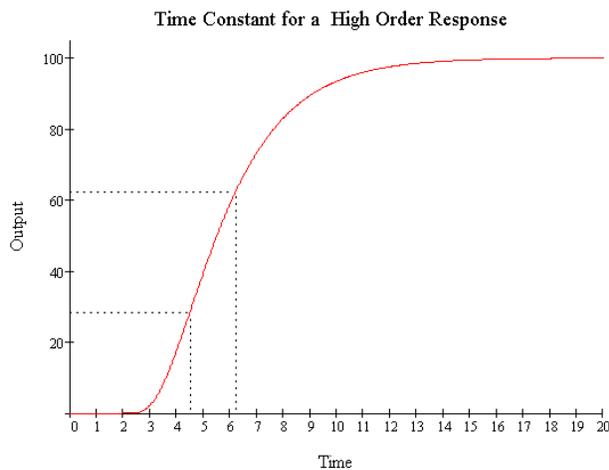
On the other hand, methods #2 and #3 both provide a good fit to the original response. However, it is not necessarily the degree of fit with which we are concerned. What we need is an accurate approximation of the system time constant and dead time. Note in Table 1 that method #2 approximates a value of time lag somewhat larger than method #3 while the dead time is somewhat smaller than that of method #3. When trying to approximate these constants, it is in fact better to approximate the dead time on the low side and the time lag on the high side. This does imply method #2 is a better method of finding these constants. On the other hand, research has indicated that perhaps method #3 provides more consistent results. In a future chapter regarding loop tuning, we will compare how each of these methods impact the tuning process.



a) Fit #1



b) Fit #2



c) Fit #3

### Method #1

- i. Draw a horizontal baseline at the initial value of the response curve (0%)
- ii. Draw a horizontal baseline tangent to the final value of the response curve (100%)
- iii. Draw a tangent line to the steepest part of the response curve allowing it to intersect with both baselines
- iv. The time from the start of the step input to the lower point of intersection represents the end of the effective system dead time (3.30s)
- v. The time between the upper and lower points of intersection represents system time lag (4.34s)

### Method #2

- i. Draw a horizontal baseline at the initial value of the response curve (0%)
- ii. Draw a tangent line to the steepest part of the response curve allowing it to intersect with the baseline
- iii. Determine that point at which the response reaches 63.2% of its total change and draw a vertical line to the corresponding time.
- iv. The time from the start of the step input to the lower point of intersection represents the end of the effective system dead time (3.30s)
- v. The time between this point of intersection and the time found in step iv represents system time lag (2.90s)

### Method #3

- i. Determine the time at which the system reaches 63.2% of total response (6.32s)
- ii. Determine the time at which the system reaches 28.3% of total response (4.52s)
- iii. The time constant is 1.5 times the difference between the times found in steps i and ii (2.70s)
- iv. The dead time is the difference between the time found in step i and the time constant calculated in step iii (3.62s)

Figure 4: Analyzing a high order response

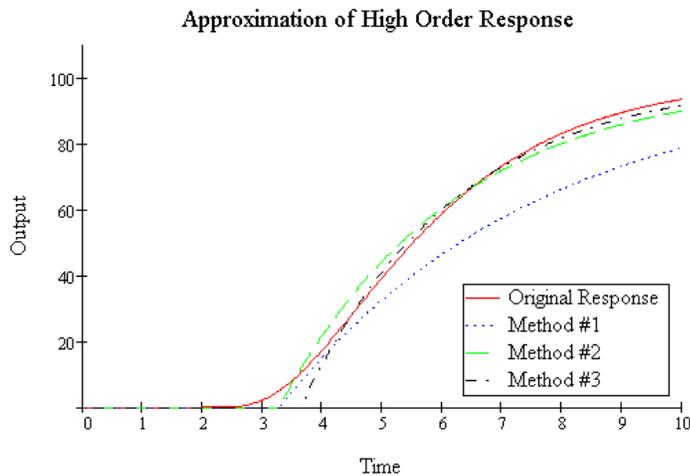


Figure 5: Comparison of models to estimate high order response

<b>Method</b>	<b>Time Lag</b>	<b>Dead Time</b>
#1	4.34 s	3.30 s
#2	2.93 s	3.30 s
#3	2.70 s	3.62 s

Table 1: Comparison of time elements derived from the various methods for analyzing a high order response

### Response of a valve

In a previous section, we discussed the performance and selection characteristics of control valves. However, control valves can also exhibit time delay and dead time. These elements manifest themselves in the form of valve sticking (sticktion) and deadband. All valves, even new valves, exhibit some degree of sticking and deadband. It is the quality of the valve (when new), and the maintenance of the valve (when installed), that determines the degree to which these phenomena are manifested.

Valve sticking is the result of static friction forces between the valve stem and valve packing. It is often referred to as sticktion, short for sticky friction. Once the valve is placed in service, these frictional forces may vary for any number of reasons. If the fluid being controlled has lubricating properties, it is conceivable one may see a reduction in friction. On the other hand, corrosive fluids may begin to attack the valve stem or packing gland causing an increase in sticktion. Over tightening the packing gland nut will also impact the observed degree of sticktion.

Sticktion will appear as a jerky movement of the valve stem. When a signal is applied to the valve, it may be inadequate to initially overcome the frictional forces between the valve stem and packing. This means the controller sees no feedback regarding the correction it just made. The controller will continue to increase the signal at the same rate of change. As the signal increases to a point where it overcomes the frictional forces, the valve stem jumps. However, it can only move a certain amount before frictional forces again exceed the force applied due to the control signal, thus preventing the valve stem from moving. Figure 5 illustrates such movement.

This curve shows the result of a valve cycling between 27.5% and 72.5% of total valve movement. The smooth curve represents the signal applied to the valve. The staircase

trace is the resulting valve movement. Sticktion is generally stated as a percentage of the total valve movement. The valve in Figure 5 exhibits sticktion of 2%.

Valve deadband is the result of dynamic frictional forces between the valve stem and the valve packing. Deadband shows up when the valve must change direction. It is a measure of the amount of change in the applied signal that must occur before the valve physically changes position. Refer again to Figure 5. When the cycle reaches its peak, the signal to the valve reaches a value of 72.5%, however, the valve is only at 69.5%. As the signal reverses, it must drop to a value of 67.5% before the valve changes position. In other words, the valve deadband is  $72.5 - 67.5 = 5\%$ .

Note that both the deadband and the sticktion will appear to the controller as a form of dead time. In the case of signal reversal, there is approximately an 8 second span over which the control valve does not move even though the signal to the control valve is changing. Similarly, each stair-step movement of the valve stem takes approximately 1.3 seconds. Ultimately, this affects the tuning of the controller.

Valves also exhibit a time lag. Figure 6 shows a valve response with a valve exhibiting 5% deadband and 2% stick-slip. The solid trace is a first order approximation of the change in valve position. If one were to apply the methods of finding a time constant as depicted graphically in Figure 2, one would find a time constant for this valve somewhere between three and four seconds. Note that sticktion prevents us finding a more accurate value of time lag.

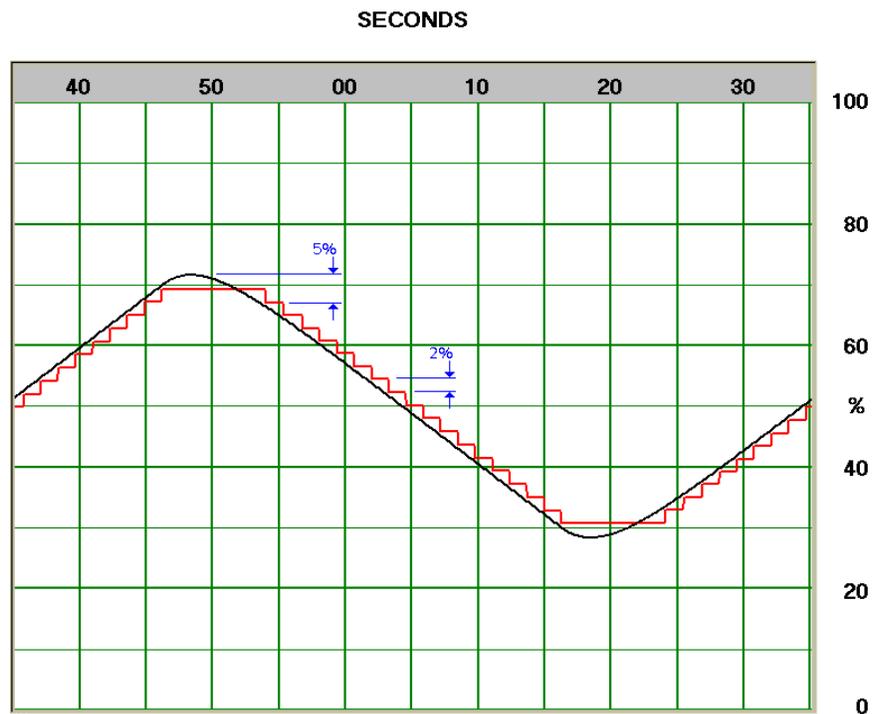


Figure 6: Valve stick and deadband as may be observed in a cycling valve. This trace is a screen shot of a valve simulation from PC-ControlLab. Valve stick is 2% and deadband is 5%.



Figure 7: An approximation of the first order response of a valve exhibiting sticktion and deadband. The time constant for this valve is between three and four seconds.

### ***Time constants for physical systems***

Physical systems can take many forms. In this discussion, we will address electrical systems, fluid systems, and thermodynamic systems. All elemental devices have a time constant associated with it. Mathematically, we can show such a time constant is a function of a resistance to energy flow and the capacity to store energy. At this point, it becomes a matter of how we define these two quantities for individual systems.

### **Electrical Systems**

A typical RTD temperature transmitter is shown in Figure 8. At the risk of being overly simplistic, such devices can often be reduced to an equivalent RLC (resistance, inductance, capacitance) circuits. Although such circuits may be series, parallel or series - parallel, circuits, our discussion will concentrate on a series circuit as indicated in Figure 9. Such circuits contain two energy storage devices, a capacitor and an inductor, in series with a resistance, and will exhibit a second order response similar to that shown in Figure 4. To understand the time constant of a high order system, we must first understand first order electrical systems in the form of an elementary RC circuit, as shown in Figure 10, and an elementary RL circuit as shown in Figure 11.



Figure 8: Temperature Transmitter  
(Courtesy KMC Controls)

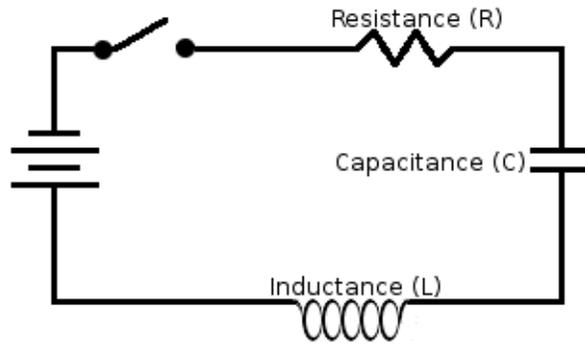


Figure 9: An electrical circuit can be modeled as one with Resistance, Capacitance and Inductance

From a course in basic electrical circuits, we know the following:

- Quantity of electrical energy ( $q$ ) is measured in Coulombs as an electrical charge.
- The flow of electrical energy is measured in amperes ( $I$ ). An ampere is the flow of one coulomb ( $q$ ) of electrical charge per second.
- The flow of current is the result of an electromotive force measured as voltage ( $E$ ). It is this electrical 'pressure' that forces an electrical current to flow through an electrical resistance.
- Ohm's law states that voltage is the product of current and resistance. If we rearrange this equation, we can define resistance as voltage divided by current.
- A capacitor is used to store electrical charge. The total capacitance of a capacitor, which is measured in farads, is a function of voltage and total stored charge. The greater the voltage across the capacitor, the more charge it will hold. Mathematically, charge is expressed as the product of capacitance and voltage. Rearranging, we can express capacitance as charge per unit of voltage.
- An inductor is a device comprised of several coils of wire that opposes a change in current. This opposition is in the form of an induced EMF. In other words, as current in a circuit increases, the inductor generates an opposing EMF. The total inductance of an inductor, which is measured in henries, is a function of the generated voltage and the observed change in current. Mathematically, the EMF generated is expressed as the product of inductance and the change in current. Rearranging, we can express inductance as voltage per unit change of current.

Assume the RC circuit in Figure 10 is unpowered and the capacitor has no stored charge. A digital voltmeter is used to measure the voltage across the capacitor as shown. Closing the switch constitutes a step input to the circuit; an instantaneous increase from zero volts to battery voltage. The capacitor prevents the voltmeter from seeing the full battery

voltage immediately. Rather, the capacitor begins to store charge and the voltage across the capacitor will increase in a predictable manner. The response of this system will be identical to the first order response shown in Figure 2.

It was stated earlier a system time constant is the product of resistance and capacitance. Using the basic definitions described above, we can write:

$$E = I * R \text{ therefore } R = E/I$$

$$q = C * E \text{ therefore } C = q/E$$

$$\text{Since } \tau = R * C, \text{ then: } \tau = E/I * q/E = q/I = q / (q/\text{sec}) = \text{sec}$$

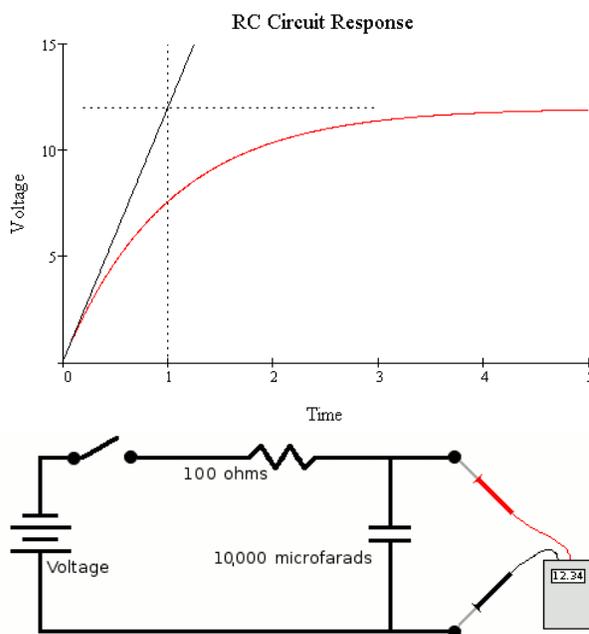


Figure 10: A simple RC circuit. The response of such a system is shown in the graph. Note the time constant is the product of resistance and capacitance.

Thus an RC circuit with a capacitance of 10,000  $\mu\text{F}$  and a resistance of 100  $\Omega$  will have a time constant of  $0.01 \text{ F} * 100 \Omega = 1 \text{ second}$ .

The time constant of a RL circuit is analyzed in a similar fashion. Assume the circuit in Figure 11 is unpowered. A digital voltmeter is used to measure the voltage across the inductor. By closing the switch, we impose a step voltage input to the system. Since an inductor opposes a change in current, initial current will be zero while the initial voltage across the inductor is equal to system voltage. As current begins to increase, the voltage across the inductor decreases. This occurs predictably in accordance with the system time constant. The response of this system will be a first order response similar to that shown in Figure 11.

The time constant of an RL circuit is the quotient of inductance and resistance. We can show this through the following mathematical relationships.

$$E = L \cdot \Delta I = L \cdot (I/\text{sec}) \text{ therefore } L = (E \cdot \text{sec})/I$$

From above,  $R = E/I$

Since  $\tau = L/R$ , Then  $\tau = [(E \cdot \text{sec})/I] / (E/I) = \text{sec}$

Thus an RL circuit with an inductance of 100,000 mH and a resistance of 100  $\Omega$  has a time constant of 100 H / 100  $\Omega = 1$  sec. The response of an RL circuit is shown in Figure 11.

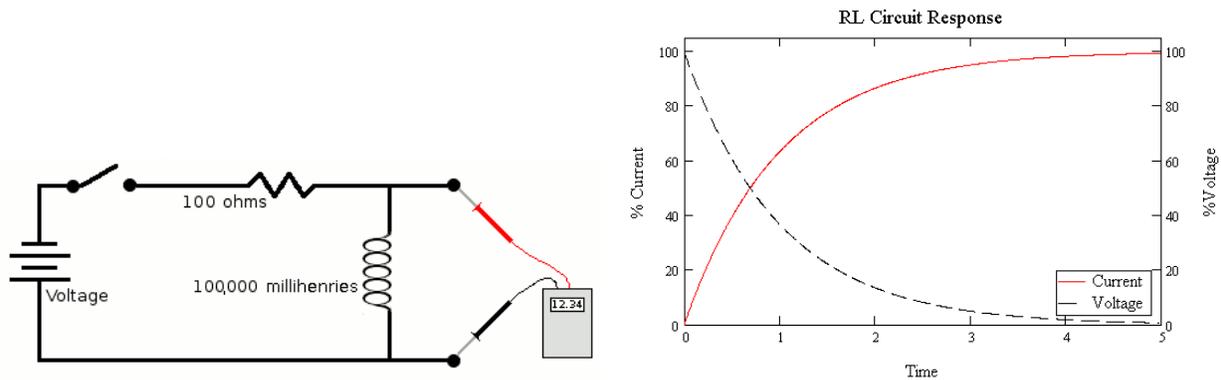


Figure 11: A simple RL circuit. Note the time constant is the quotient of inductance and resistance. Since an inductor resists a change in current, the plot used to determine the time constant is %current vs. time. We can determine the time variable voltage across the inductor by applying ohms law.

## Fluid Systems

The control of fluid systems is commonplace in HVAC and process control. Analogous to electrical systems, the time constant of a fluid system is the product of hydraulic resistance and hydraulic capacitance. To determine these quantities, let us investigate the flow of fluid through a process tank. Consider the process tank shown in Figure 12. This tank has a cross sectional area of 'A' and a fluid head equal to 'H'

From a basic course in fluid mechanics, you will remember the continuity equation states:

$$Q = V \times A$$

$Q$  = Volume flow rate (ft<sup>3</sup>/sec)

$V$  = Flow velocity (ft/sec)

$A$  = Area of flow conduit (ft<sup>2</sup>)

Energy is required for fluid to flow through the nozzle at the base of the tank. This energy is a conversion of the potential

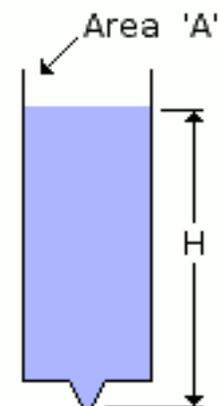


Figure 12 Process Tank

energy stored in the tank to flow energy. Potential energy is equal to the height of the fluid in the tank. Flow energy is a function of the flow velocity. Thus:

$$H_v = \frac{V^2}{2g}$$

$H_v$  = Velocity head. In this case, it is equal to total fluid head, H

Rearranging:

$$V = \sqrt{2gH}$$

Substituting into the continuity equation yields:

$$Q = Ax\sqrt{2gH}$$

This equation is known as Torcelli's theorem. It is the theoretical flow rate through a nozzle of area 'A' given a fluid potential of 'H'. However, certain phenomena will result in a somewhat lesser flow rate. In order to account for these losses, we will multiply the theoretical quantity by a discharge coefficient,  $C_d$ .

$$Q = C_d \times Ax\sqrt{2gH}$$

Now let us consider the pressure drop through any passive hydraulic element such as a valve or nozzle. Such pressure drop is defined as:

$$\Delta H = R \times Q^n \times S.G.$$

$\Delta H$  = Pressure drop (in feet of fluid head)

Q = Volume flow rate

R = Hydraulic resistance

n = Parameter to handle turbulent flow

S.G. = Fluid specific gravity

Considering the fluid level in the tank of Figure 12 and making the following assumptions:

- Consider the variation in fluid level in the tank as a flow. This flow is laminar, thus the 'n' parameter in the above equation is equal to 1
- The  $\Delta H$  term is the differential head between some value of fluid head, H, and zero head,  $H_0$ , which occurs with an empty tank. Thus  $\Delta H = H - H_0 = H - 0 = H$
- Assume the fluid has a specific gravity of 1

Then:

$$H = R \times Q$$

Note the similarity to Ohm's law. Rearranging, we can define the general form for hydraulic resistance as:

$$R = \frac{H}{Q}$$

However, there is one significant difference between a fluid system and an electrical system. Since head (H) and flow (Q) can vary with time, hydraulic resistance will also vary with time. As such, we must determine the hydraulic resistance at any instant in time. We can do so by differentiating head with respect to flow.

$$R = \frac{dH}{dQ}$$

$$H = \frac{Q^2}{C_d^2 \times A^2 \times 2g}$$

$$R = \frac{dH}{dQ} = \frac{Q}{C_d^2 \times A^2 \times g}$$

By substituting Toricelli's equation for Q:

$$R = \frac{C_d \times A \times \sqrt{2gH}}{C_d^2 \times A^2 \times g} = \frac{\sqrt{2gH}}{C_d \times A \times g}$$

Multiply the top and bottom by  $\sqrt{2gH}$  and simplify:

$$R = \frac{2H}{C_d \times A \times \sqrt{2gH}}$$

Note the denominator is equal to Q, so instantaneous hydraulic resistance can be defined as:

$$R = \frac{2H}{Q}$$

Now that resistance is defined, we need to define capacitance in a hydraulic system. Referring again to the tank in Figure 12, we know the total capacity of the tank is the volume of the tank. Taking the area of the tank in square feet and fluid head in feet, the volume of the tank, in cubic feet, is calculated as:

$$V = H \times A$$

By making an analogy to electrical systems, tank volume can be considered the [quantity of] charge on a capacitor and fluid head is analogous to voltage [potential]. We

also know that electrical capacitance is charge divided by voltage. Rearranging the above equation:

$$A = \frac{V}{H}$$

Thus the area of a tank in a fluid system is analogous to electrical capacitance. Thus, the time constant of a tank is equal to hydraulic resistance multiplied by hydraulic capacitance. Dimensionally:

$$\tau = \frac{2H}{Q} \times A = \frac{\text{feet}}{\text{feet}^3/\text{second}} \times \text{feet}^3 = \text{seconds}$$

**Example:**

A process tank experiences an inflow of 80 gpm and an outflow of 160 gpm. The tank has a diameter of 8 feet and a fluid head of 5 feet. Determine the time constant.

The net outflow is:

$$Q = 160 - 80 = 80 \text{ gpm}$$

$$80 \frac{\text{gal}}{\text{min}} \times \frac{\text{min}}{60 \text{sec}} \times \frac{\text{ft}^3}{7.48 \text{ gal}} = 0.178 \frac{\text{ft}^3}{\text{sec}}$$

The area of the tank is:

$$A = \pi 4^2 = 50.2 \text{ ft}^2$$

The system time constant is then equal to:

$$\tau = \frac{2 \times 5 \text{ ft}}{0.178 \frac{\text{ft}^3}{\text{sec}}} \times 50.2 \text{ ft}^2 = 2823 \text{ seconds} = 47 \text{ minutes}$$

**Thermal Systems**

Another common process in HVAC control is that of heat transfer. As with electrical and fluid systems, the time constant of a heat transfer process is also the product of a resistance and a capacitance. From a basic course in heat transfer, we know that the heat absorbed by any material is defined by:

$$Q = mc_p \Delta T$$

$Q$  = Total energy transfer in BTU (Quantity)

$m$  = Mass of material in lb<sub>m</sub>

$c_p$  = Specific heat of material in BTU/(lb<sub>m</sub> F) (Heat Capacity)

$\Delta T$  = Temperature difference (Potential)

Similarly, the heat transfer through a solid material is:

$$Q = \frac{1}{R} A \Delta T$$

$Q$  = Heat transfer rate in BTU per unit time (Quantity)

$A$  = Area of surface through which heat is transferred

$R$  = Resistance to heat transfer

$\Delta T$  = Temperature difference (Potential)

If we let  $R_o = R/A$ , where  $R_o$  is the overall resistance to heat transfer, then we can rearrange and write:

$$R_o = \frac{\Delta T}{Q}$$

Thermal resistance of many materials is well known and tabulated. For example, the ASHRAE Handbook of Fundamentals tabulates the thermal resistance of many common building materials. These material properties may be expressed as one of four values:

- Resistance – Commonly referred to as R-value, this property represents the number of hours it takes for one BTU of energy to pass through one square foot of area with a one-degree temperature difference for a given thickness of material.

$$R \equiv \frac{ft^2 \text{ } ^\circ F \text{ } hr}{BTU}$$

- Resistance per inch – This is R-value per inch

$$r \equiv \frac{ft^2 \text{ } ^\circ F \text{ } hr}{BTU \text{ } in}$$

- Conductance – Thermal conductance is the reciprocal of thermal resistance. It represents the number of BTU's that will pass through one square foot of area within one hour given a temperature differential of one degree for a given thickness of material.

$$C \equiv \frac{BTU}{ft^2 \text{ } ^\circ F \text{ } hr}$$

- Conductivity – Thermal conductivity is the reciprocal of R-value per inch.

$$k \equiv \frac{BTU \text{ } in}{ft^2 \text{ } ^\circ F \text{ } hr}$$

Given all of the above definitions, we can now define total thermal resistance and total thermal capacitance of a system. Since overall resistance to heat transfer,  $R_o$ , is simply equal to R-value divided by area, then:

$$R_o = \frac{R}{A} = \frac{\frac{ft^2 \text{ } ^\circ F \text{ hr}}{BTU}}{ft^2} = \frac{^\circ F \text{ hr}}{BTU}$$

Total capacitance is equal to the specific heat of the material multiplied by the total quantity (mass) of material.

$$C = c_p \times mass = \frac{BTU}{lb_m \text{ } ^\circ F} \times lb_m = \frac{BTU}{^\circ F}$$

Then the time constant of a thermal system is:

$$\tau = R \times C = \frac{^\circ F \text{ hr}}{BTU} \times \frac{BTU}{^\circ F} = \text{hour}$$

### Example:

A finned-tube hot water coil is fabricated of aluminum tubes and fins. The total external heat transfer area is 60 ft<sup>2</sup>. The total mass of aluminum of which the coil is fabricated is 200 lb<sub>m</sub>. The overall conductivity of the coil is 8.45 BTU/(hr ft<sup>2</sup> °F). What is the time constant for this coil?

From handbook data,  $c_p$  for aluminum is 0.214 BTU/lb<sub>m</sub> °F

$$C = 0.214 \frac{BTU}{lb_m \text{ } ^\circ F} \times 200 lb_m = 42.8 \frac{BTU}{^\circ F}$$

$$R_o = \frac{\left[ 8.45 \frac{BTU}{hr \text{ ft}^2 \text{ } ^\circ F} \right]^{-1}}{60 \text{ ft}^2} = 0.00197 \frac{^\circ F \text{ hr}}{BTU}$$

$$\tau = R \times C = \left[ 0.00197 \frac{^\circ F \text{ hr}}{BTU} \right] * \left[ 42.8 \frac{BTU}{^\circ F} \right] = 0.0844 \text{ hour} = 5.1 \text{ minutes}$$

### Putting it together

It is not unusual to get lost in the mathematics of the above material and not realize the physical and practical importance of what it means to control system design, operation, and maintenance. Although there are definitely situations in which the calculation of the time constants of each component of a process system is necessary, the majority of situations do not require one to do so. However, it is quite useful to be able to determine which component of the process loop is likely to be the dominant time lag or

dominant dead time. The above mathematical presentations are provided to aid you in identifying those components that may be dominant. In an effort to provide such insight, consider the following scenarios.

All too frequently, the application of any single control device to a given process is often considered only in the case of steady-state operation. For example, a previous section discussed the application of temperature sensors. Assume an application is controlled by sensing the temperature of some flowing fluid. If the system is at steady-state conditions, does it really matter if the temperature sensor is a highly responsive device with a time constant of only seconds, or a massive element in an oil-filled pipe well with a time constant measured in minutes? In the case of steady-state operation, it really doesn't matter. But we don't provide control to handle the steady-state scenario. Control is necessary to maintain the process variable at a fairly constant value during otherwise dynamic conditions. It becomes apparent that if the control process had to be very responsive, then we would want to take the time to select a responsive (short time constant) sensing element.

In a similar vein, certain types of liquid-to-liquid heat exchangers are more responsive than others. For example, plate and frame heat exchangers tend to have a somewhat shorter time constant than a shell and tube exchanger. Such a consideration may or may not be important to the design of the system.

On the other hand, consider two process tanks as shown in Figure 13. Each tank has the same outflow,  $Q$ , and the same liquid head,  $H$ , thus each has the same hydraulic resistance. However, they each have a different capacitance as measured by their areas. Assume you are to control the level in each tank. Which tank would you suppose to be more sensitive to variation in inflow and outflow? If you said the smaller tank, you would be correct. This is rather self-evident when one realizes that the smaller area means there is a smaller volume of liquid for every foot of liquid head. Thus, the fluid level will drop further and faster for a given outflow than with the larger tank. This is born out by the fact the smaller capacitance also means a smaller time constant for the tank. This means the tank is more responsive to variation in flow. As will be seen in the chapters on tuning, this will have definite impact on control loop tuning.

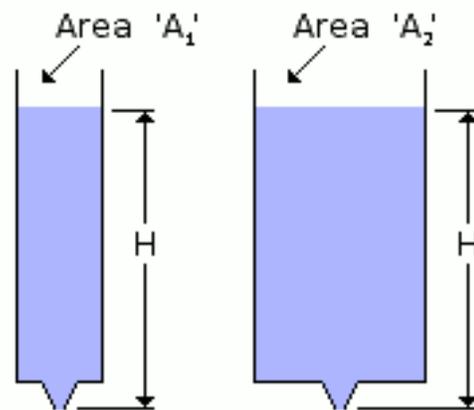


Figure 13 Two process tanks of equal outflow, equal head, but unequal capacitance