Process Gain and Loop Gain

By now, it is evident that one can calculate the sensitivity for each component in a controlled process. Sometimes, this sensitivity is referred to as a gain. The confusion is understandable since both sensitivity and gain are defined as the ratio of the change in output for a unit change in input. The primary difference between sensitivity and gain is the use of units. When calculating gain, the input and output signals are expressed in the same units, usually a control signal, so the final value of gain is unit-less. However, sensitivity usually involves expressing the same ratio in terms of engineering units. This statement is best demonstrated by determining the sensitivity and gain of a given controller. Consider a control loop that controls the outlet temperature on the shell side of a shell and tube heat exchanger by manipulating the flow rate of hot water on the tube side. The sensor is a 100 to 200 ^oF RTD with an output of 4 to 20 ma. The controller sends a 4 to 20 ma signal to an electronic valve actuator. Assume a throttling range of 25 ^oF. The controller sensitivity is then:

$$CS = \frac{\Delta Output}{\Delta Input} = \frac{(20-4)\,ma}{25^oF} = 0.64\frac{ma}{^oF}$$

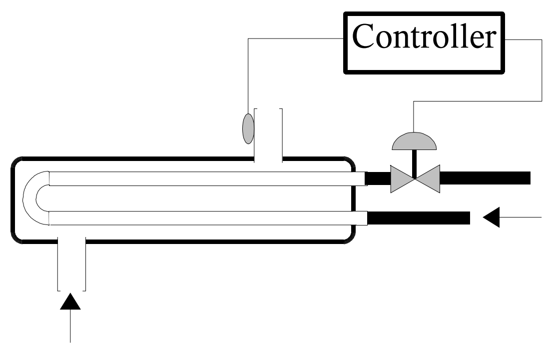

Figure 1 Control of a heat exchanger

Note that the input signal is expressed in terms of an engineering unit while the output signal is expressed in terms of a control signal. To determine controller gain, we must express both input and output in terms of the same signal. We can do this by expressing these quantities as control signals. In this case, we will express the input as a current (ma) instead of temperature. This requires we find the sensitivity of the sensor.

$$sensor\ sens = \frac{(20-4)\,ma}{(200-100)\,^oF} = 0.16\frac{ma}{^0F}$$

Once the sensitivity is known, the throttling range of 25 ^oF, which represents controller input, can be expressed as:

$$0.16\frac{ma}{^oF}\times 25^oF = 4\,ma$$

Then controller gain is:

$$gain = K_c = \frac{\Delta Output}{\Delta Input} = \frac{(20-4)ma}{4\,ma} = 4$$

Sensitivity around the loop

Just as we can express controller performance or component performance in terms of a sensitivity or gain, we can also express the performance of a controlled process in terms of a gain or sensitivity. This is referred to as process gain. The product of this process gain and the controller gain is referred to as loop gain. Let's look at an example.

Depicted below is a block diagram of the above shell and tube control loop. Consider the portion of the loop from point 1 clockwise around the loop to point 2. The control element is the valve actuator and control valve. The manipulated variable is the flow rate through the tube-side of the heat exchanger. The process is the heat exchange between the shell side fluid and the tube side fluid. Physically, it consists of the heat exchanger itself, the fluid flowing through the exchanger, the temperature of the fluids and the flow rates of the fluids. The controlled variable is the temperature of the fluid leaving the exchanger on the shell side. The sensor is likely an RTD, thermistor, or thermocouple. The sensor connects to a transmitter (a signal conditioner), which translates the millivoltage or resistance signal from the sensor to a signal capable of being received by the controller; usually a voltage, current or pneumatic signal. Each of these elements can be assigned some value of sensitivity, the product of which is the process gain. Mathematically:

$$process\ \ gain = K_p = \frac{Change\, in\ process\, output\ signal\ at\ point\, 2}{Change\, in\ process\, input\ signal\ at\ point\, 1}$$

$$controller\ \ gain = K_c = \frac{Change\, in\ process\, output\ signal\ at\ point\, 1}{Change\, in\ process\, input\ signal\ at\ point\, 2}$$

$$Loop\, Gain = K_L = K_p \times K_c$$

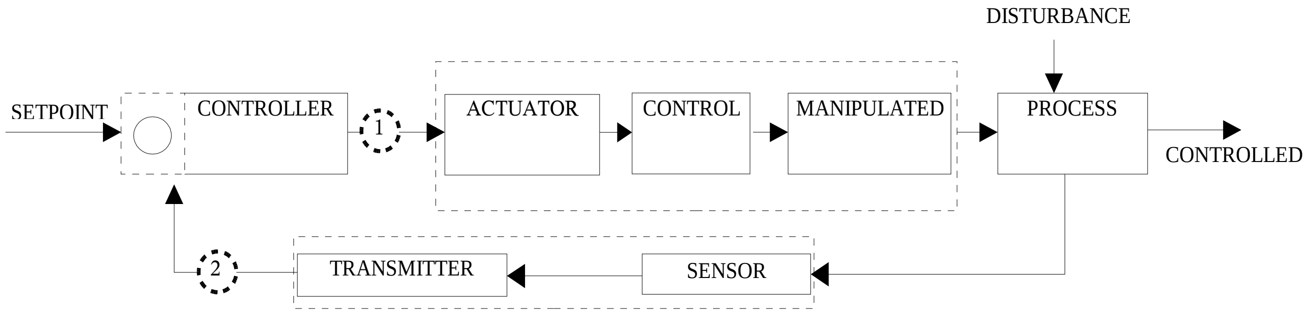

Figure 2 Block diagram of a shell and tube heat exchanger

Let's assume the process heat exchanger, a nominal 6,000 Mbtuh unit, has the following design characteristics:

Tube side:

Flowrate: 0 to 350 gpm
Inlet Temp: 240 ^oF

Shell side:

Flowrate: 350 gpm (nominal)
Inlet Temp: 130 ^oF
Outlet Temp: 160 ^oF

Let us also assume the following conditions:

- Actuator: A 4 to 20 ma signal moves the actuator through a full stroke of 2.50". This results in a sensitivity of 2.50 in / 16 ma = 0.15625 in/ma
- Valve: A linear valve with a 2.50" stroke controlling a flow from 0 to 350 gpm. Sensitivity is equal to 350 gpm / 2.5 in = 140 gpm/in
- Process: Heat exchanger as defined above. Assume a sensitivity of 0.1174 ^oF/gpm.
- Sensor: A 1000 Ω RTD with a resistance of 1146.84 to 1359.70 Ω over a temperature range of 100 to 200 ^oF. The sensitivity is 2.1286 Ω/^oF
- Transmitter: A 4 to 20 ma output signal with an input resistance of 1146.84 to 1359.70 Ω. Transmitter sensitivity equals 0.07516 ma/Ω

Sensitivities are multiplicative. We can determine the sensitivity of the loop, that is, the loop gain, as follows:

$$K_p = 0.15625\frac{in}{ma}\times 140\frac{gpm}{in}\times 0.1174\frac{^oF}{gpm}\times 2.1286\frac{\Omega}{^oF}\times 0.07516\frac{ma}{\Omega} = 0.411\frac{ma}{ma}$$

This means if the signal to the valve actuator changes by one milliamp, (process input), you will see a change of 0.411 milliamps at the output of the sensor/transmitter at the other end. If the controller has a gain of 4, then we can determine the loop gain as:

$$K_L = K_p \times K_c = 0.411 \times 4 = 1.64$$

This means that for a signal change of 1 ma to the input terminals of a process controller, the signal will travel around the loop and be amplified by a factor of 1.64. The controller will then see a change at it's input terminals of 1.64 ma as feedback.

It is worth noting the above analysis is quite simplified. The primary reasons deal with the nonlinearities of certain components of the control loop as follows.

- The control valve was assumed a linear valve. In general, a linear valve is inappropriate for the control of a heat exchanger. In practice, the installed valve is likely an equal percentage valve. A possible exception to this is the control of very high temperature process water (400+ ^{o}F) with a large ΔT across the exchanger (100 – 200 ^{o}F). Under such circumstances, the heat exchanger exhibits near linear characteristics and a linear valve may be appropriate.
- This brings up the second point. A sensitivity was provided for the heat exchanger over a given range. This sensitivity was based upon the premise the output of the heat exchanger is linear. Again, this is not the case. Most heat exchangers exhibit extreme nonlinear characteristics.
- Another source of nonlinearity is the temperature sensor. Temperature sensors are not generally linear devices. However, the above example was based upon the use of an RTD. As discussed during the material on sensors and transmitters, an RTD does exhibit nearly linear characteristics. However, if the sensor was a thermocouple or a thermistor, these devices exhibit a greater degree of nonlinearity. As such, it is necessary that a properly designed transmitter is used to compensate for these nonlinearities.
- All control loops and controlled processes exhibit any number of dead times and time delays. Although these time elements do not affect the sensitivity of any given device, it has a great impact on the response of the control loop.

Variable Loop Gain

We covered the variability of installed valve characteristics and heat exchanger characteristics at the beginning of the quarter. As was discussed at the time, the ideal is to match the valve to the heat exchanger perfectly. This would result in a linear heat exchanger output relative to valve stem movement. However, improperly sized control valves or the necessity to install 'off-the-shelf' valves that do not match the characteristics of the heat exchanger may result in nonlinearities. Also, it is quite difficult to linearize the output of a steam coil. The end result might be something as shown in Figure 3.

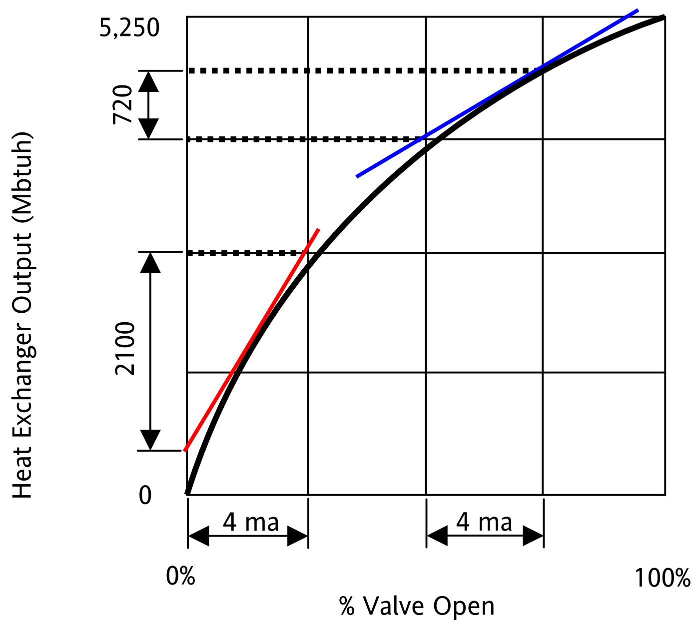

Figure 3 Illustration of variable process gain

The graph indicates a valve accepting a 4 to 20 ma signal to stroke full open to full closed. The heat exchanger is designed for an output of 0 to 5,250 Mbtuh over this range. The heat output is not linear relative to valve position. If we look at two specific points, namely 70% valve lift and 15% valve lift, we find very different values of sensitivity (gain). At the lower range, the sensitivity is about 2100/4 = 525 Mbtuh/ma while at the upper range, the sensitivity is about 720/4 = 180 Mbtuh/ma. This represents almost a threefold variation in loop gain if the process operated between these extremes. Needless to say, if we have multiple elements in the control loop exhibiting such variation in sensitivity, the process will become very difficult, if not impossible, to tune properly. If the process is properly tuned at that point where the loop exhibits high loop gain, the control system will be sluggish at the other extreme. Conversely, if the loop is tuned to respond properly at that point where the loop exhibits a lower degree of gain, the loop will likely be over aggressive, even unstable, at the opposite end.